

Auctions

Auction:

- One seller and a small number of potential buyers

The mirror image –

Contract auction / Procurement auction:

- One buyer and a small number of potential sellers.
- The buyer decides on the purchasing procedure, potential sellers bid their prices.

Monopsony:

Buyer determines price,
sellers choose quantities given this price

When are auctions used?

- A unique object
 - well defined? indivisible?
- Uncertainty about who should get the object / the contract
- Uncertainty about the object's value / the project costs
- Commitment to selling / buying procedure

Alternatives to auctions

Market

- decides who gets the object / project
- but how to determine the price?

Bargaining

- determines the price
- but how to determine who is the counterpart?

Handing out for free

- beauty contest
- lobbying costs

Two concerns with an auction

- For society - *efficiency*: Is the object bought by the bidder with the highest willingness to pay?
- For the seller: Is the price the highest possible?

Several auction procedures

How are these questions affected by the procedure chosen?

Various kinds of auctions

- Sealed bids vs. open bids

- Open bids:
 - ascending bids – English auction
 - bidders submit higher and higher bids until only one bidder remains
 - art, collectibles

 - descending bids – Dutch auction
 - seller starts with a high price and cries out lower and lower prices until a bidder accepts
 - flowers (Netherlands), fish (Israel), tobacco (Canada)

- Sealed bids
 - First price:
 - The bidder with the highest bid wins and pays his bid.
 - real estate, government procurement

 - Second price:
 - The bidder with the highest bid wins and pays an amount equal to the second highest bid.
 - Vickrey auction [Vickrey, *J Finance* 1961]
 - William Vickrey, Nobel laureate 1996
 - stamps etc. [Lucking-Reiley, *J Econ Perspectives* 2000]

Basic model

- Bidders are risk neutral
- Bidders' valuations are different but independent
- Each bidder knows only his own valuation
- Seller doesn't know any bidder's valuation
- No observable differences among the bidders
- Reservation price?

Bidder behaviour

(i) English auction

- continuing bidding is profitable as long as own valuation $>$ current high bid
- this strategy is independent of what other bidders do (dominant strategy)
- the winner is the one with highest valuation
→ efficiency
- price is (just above) second highest evaluation

(ii) Sealed-bid second-price auction

bidder B's valuation = v

bidder B's bid = b

largest bid from others = a

- With a valuation of v , what should be bidder B's bid, b ?
Distinguish between two cases:
 - $a > v$: B's decision does not matter
 - $a < v$: B wins if $b > a$, and earns $(v - a)$
- Bidding $b < v$ reduces B's chances to win but does not affect what he has to pay if he wins.
- Optimum bid: $b = v$
(dominant strategy)
- The winner is the one with highest valuation
→ Efficiency
- The price equals second-highest valuation
- English auction and sealed-bid second-price auction are equivalent with respect to winner and price.
- Contract auction:
 - winner is the one with lowest cost
 - price equals second-lowest cost
- Calculating the bid is easy

(iii) Sealed-bid first-price auction

- Bidder trades off two concerns:
Bidding $b < v$
 - reduces his chances to win; not good.
 - reduces the price he has to pay if he wins; good.
- This trade-off makes the optimum bid lower than v .
- The bidder knows that other bidders think the same way:
All bidders bid below their valuation. This makes the optimum bid even lower.
- This also holds for (iv) Dutch auction
- The winner is the one with the highest valuation
- The price equals highest bid, which is lower than highest valuation
- Expected price = Expected second-highest valuation
- Calculating bid is difficult

Equilibrium bid – sealed-bid first-price auction

n bidders, $v_i \in [v_l, v_h]$, $i \in \{1, \dots, n\}$

cumulative distribution function: $F(v_i)$, $i \in \{1, \dots, n\}$

Let's focus on a symmetric equilibrium. Bidders are not identical, in that they have identical valuations. But there are no observable differences, so their valuations are all drawn from the same cdf.

In a symmetric equilibrium, there exists some function $B(v)$, which is the same for all players, so that if one's valuation is v , the equilibrium bid is $B(v)$.

Consider bidder i . He does not know the other bidders' v s but believes that their bids depend on their valuations according to the function $B(v)$. Assume: $B' > 0$.

\Rightarrow A bid of b implies a valuation equal to $B^{-1}(b)$.

The probability that i 's bid b_i is the winning bid =
 $[F(B^{-1}(b_i))]^{n-1}$

Bidder i 's expected profit:

$$\pi_i = [v_i - b_i][F(B^{-1}(b_i))]^{n-1}$$

Optimum bid satisfies: $\frac{\partial \pi_i}{\partial b_i} = 0$

$$\Rightarrow \frac{d\pi_i}{dv_i} = \frac{\partial \pi_i}{\partial v_i} + \frac{\partial \pi_i}{\partial b_i} \frac{db_i}{dv_i} = \frac{\partial \pi_i}{\partial v_i} = [F(B^{-1}(b_i))]^{n-1}$$

In a symmetric equilibrium: $b_i = B(v_i), \forall i. \Rightarrow v_i = B^{-1}(b_i)$

In equilibrium, bidders' beliefs about each other's valuations are correct.

$$\Rightarrow \frac{d\pi_i}{dv_i} = [F(v_i)]^{n-1}$$

Assume (reasonably): $\pi_i = 0$ if $v_i = v_l. \Rightarrow B(v_l) = v_l.$

Integration:

$$\pi(v_i) = \int_{v_l}^{v_i} [F(x)]^{n-1} dx$$

Two expressions for bidder i 's profit – must be equal.

$$\pi_i = [v_i - b_i][F(B^{-1}(b_i))]^{n-1} = \int_{v_l}^{v_i} [F(x)]^{n-1} dx$$

$$\Rightarrow B(v_i) = b_i = v_i - \frac{\int_{v_l}^{v_i} [F(x)]^{n-1} dx}{[F(v_i)]^{n-1}}$$

Common for all four kinds of auctions (in the base model):

- Efficiency: Object to the bidder with highest valuation (or lowest cost)
- Revenue equivalence: All four kinds give the seller the same expected income.
- An increase in the number of bidders increases the expected price.
 - the more bidders, the higher is the expected second-highest valuation.

Difference among the auctions:

- Bid more difficult to calculate in sealed-bid first-price and Dutch auctions than in sealed-bid second-price and English auction.

Seller's reservation price

Revenue equivalence in the basic model: Seller indifferent between auction procedures. But what about a reservation price?

A parallel situation: The monopolist's problem

A monopolist trades off two concerns:

- wants to sell large quantities → low price
- wants to earn a profit per unit sold → high price

Optimum trade-off: Price above marginal cost

Auction: Seller trades off the same two concerns:

- wants to sell the object → low reservation price
- wants to earn a profit if the object is sold
→ high reservation price

The two highest valuations: v_1, v_2

Reservation price: r

Three cases:

- (i) $v_1 > v_2 > r$: increasing r has no effect
- (ii) $v_1 > r > v_2$: increasing r increases the price
- (iii) $r > v_1 > v_2$: increasing r reduces the chances to sell

Optimum reservation price with 1 bidder

Bid = r or nothing

Seller's own valuation: v_0

Seller's expected profit:

$$\pi(r) = r[1 - F(r)] + v_0F(r)$$

$$\text{FOC: } [1 - F(r)] - rf(r) + v_0f(r) = 0$$

$$\Rightarrow v_0 = r - \frac{1 - F(r)}{f(r)} \equiv J(r)$$

i.e., marginal cost = marginal revenue

$$\Rightarrow r = J^{-1}(v_0)$$

Generally:

If highest bidder has valuation v , his expected gain is

$$\frac{1 - F(v)}{f(v)}$$

so that the expected price in this case is

$$v - \frac{1 - F(v)}{f(v)} = J(v)$$

The seller sells only if $J(v) \geq v_0$ for the highest bid

$$\Rightarrow r = J^{-1}(v_0)$$

Efficiency with a reservation price:

- With a reservation price, the object may not be sold, even if a bidder exists with $v > v_0$.
- *Ex-ante* efficiency vs. *ex-post* efficiency.

Some extensions

(i) Observable differences among the bidders

Example:

Public procurement – domestic vs. foreign firms.

Suppose foreign firms are more cost effective than domestic ones.

- English auction and sealed-bid second-price auction are still efficient.
- Sealed-bid first-price auction no longer efficient: it is possible to win the auction without having the lowest cost.
- It is optimum for the procurer to discriminate between bidder groups, and one is no longer certain that the project is won by the lowest-cost bidder.
- In the example: It is optimum to discriminate in favour of the domestic firms. This favouring
 - increases the chance of getting an inefficient supplier, but also
 - lowers the bid from the efficient firms

(ii) Risk-averse bidders

- In a sealed-bid first-price auction, risk-averse bidders bid *higher* than risk-neutral ones. An increase in the bid
 - (1) increases the chance of winning, and therefore getting something
 - (2) reduces what one earns in case of winning.

With risk aversion, (1) gets more important than (2)

- Contract auction: Risk averse bidders bid more aggressively than risk neutral bidders.
- The seller gains more in a sealed-bid first-price auction than in a sealed-bid second-price auction.

(iii) Correlated valuations

- Extreme case: identical valuations. Bidders do not know the object's true value but have access to different pieces of information about this value. No bidder knows what other bidders know.

- More common in auctions than in contract auctions?

Auctions:

- buying for resale
- exclusive rights

Contract auction

- pioneering projects with great cost uncertainty for all potential suppliers

- “Winner’s curse”
 - Bidders base bids in a sealed-bid auction on estimates. The bidder with the most optimistic estimate wins.
 - If you win, then you will wish to revise your estimate: The winner is the most optimistic one.
 - But this is taken into consideration in the bids: Bids are even lower because of the “winner’s curse”.

- In an English auction, bidders learn from each other during the bidding process. This reduces the winner’s-curse problem.
 - With correlated values, an English auction is preferred by the seller to the other kinds.

- Asymmetric information
 - one bidder knows the object’s true value
 - US offshore oil and gas lease auctions
 - Porter, *Econometrica* 1995

Other issues

- Collusion
 - second-price auction better for sustaining collusion among bidders than first-price auction
 - open bids better than closed bids
 - contract auctions: *Norsk Standard*
- divisible objects
 - securities, quotas
- combined bids
 - petroleum: price on exploration right + production fee
 - vague projects: price + content
- entry costs, number of bidders, participation fee
- auctioning incentive contracts
- competition *for* a market vs. competition *in* a market

Efficiency of auctions

- Which auction procedure to use?
 - revenue equivalence
 - easily calculated bids
 - sealed-bid second-price auctionBut: risk aversion? correlated values?
- Which objects are sold most effectively in an auction?
 - unique object
 - uncertainty about willingness to pay:
how large? who?
A Norwegian example.
Before: Forced auctions of apartments
Now: Forced sales of apartments through the regular market
- Does price affect efficiency?
 - one unit – no quantity effects from price change
 - divisible objects (quotas, securities): quantity effects

Repeated auctions

- Less aggressive bidding today in order not to reveal one's high valuation before future auctions (the "ratchet" effect)
 - better to have large projects? negotiating renewal with current supplier?
- Capacity constraints: The winner of a contract today may not have capacity to participate in the next round.